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Young Children's Concept of Shape: Van Hiele Visualization Level of Geometric Thinking[1]

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Abstract: *There is a renewed focus on the geometric learning of young children in today's mathematics curriculum. This paper, in the light of the results of the study by Clements, Swaminathan, Hannibal, and Sarama (1999), "Young Children's Concepts of Shape", attempted to determine the criteria used by Primary One and Primary Two children in Singapore to distinguish members of a class of shapes, namely circles, squares, triangles and rectangles. It also aimed to find out whether there was evidence of a prerecognitive level before the van Hiele Level 0 (Visualization).*

Introduction

Research focusing on children's concept of geometric shapes began in the 1950s. Psychologists Piaget and Inhelder recorded their initial observations of the developmental levels of children's geometric understanding in "The Child's Conception of Space" (Piaget & Inhelder, 1967). As well, Dutch husband-and-wife educators, Pierre van Hiele and Dina van Hiele focused their research on children's reasoning about geometric concepts. The van Hiele's proposed a Theory of Geometric Thought comprising five sequential and hierarchical levels as a means for identifying children's level of geometric maturity. Their theory is three-fold. Firstly, it comprises five levels - Level 0: Visualization, Level 1: Analysis, Level 2: Informal Deduction, Level 3: Deduction, and Level 4: Rigor. Secondly, it states that a learner must proceed through the levels in order and the learner must have acquired the strategies of the preceding levels to function successfully at a particular level. Thirdly, Phases of Learning provide a sequence for helping learners to advance from a particular level to the next.

Other research studies have shown that young children form ideas and concepts about shapes before they enter school (Clements, 2001; Clements & Sarama, 2000). They form ideas and concepts about common shapes – such as circles, squares, triangles and rectangles – from the toys, books and television programmes they come in contact with everyday (Hannibal, 1999). It is thus not surprising that they have many ideas about these common shapes by the time they start formal schooling.

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Young children's ideas and concepts, though limited, stabilize by six years of age (Clements & Sarama, 2000b). Their ideas and concepts, however, need not be totally accurate. Several studies on young children's ideas and concepts about common shapes were conducted. For example, in Clements, Swaminathan, Hannibal, and Sarama's (1999) research, they asked 97 children, ages 3 to 6, to identify circle/square/triangle/rectangle in collections of shapes on paper. Resulting from this study, Clements et al. claimed that there was evidence that a prerecognitive level exists before van Hiele Level 0 (Visualization). That is, they found that "some children appear to use both matching to a visual prototype and reasoning about components and properties to solve these selection tasks". They thus felt that this prerecognition level exists before the van Hiele's stated Level 0 (Visualization) where children judge and operate on shapes and figures according to their appearance. Consequently, Clements et al. felt that Level 0 (Visualization) should be reconceptualized as syncretic (that is, the interaction of both verbal declarative and imaginistic knowledge).

The Singapore Geometry Curriculum for Primary One and Primary Two Children

The Singapore mathematics syllabus (Ministry of Education, 2000) reflects the Ministry of Education's focus on mathematical problem solving and the vision of a "Thinking Schools, Learning Nation". The development of mathematical concepts such as geometrical concepts, skills and underlying processes thus prepare Singaporean children towards such a vision.

In Primary One, the instructional objectives for geometry expect children to sort objects by shapes and to recognize and name the four basic shapes – circle, triangle, rectangle, and square (Ministry of Education, 2000, p. 39). The geometric instructional objectives for Primary Two ask children to identify and name the following shapes that make up a given figure - square, rectangle, triangle, circle, half-circle, and quarter circle - and to identify straight and curved lines (Ministry of Education, p. 50).

Thus, at both the Primary One and Primary Two levels, teachers would normally provide children with manipulative materials, as encouraged in the Teachers' Guide, and then discuss pictures and exercises as presented in the textbooks and workbooks. Teachers might also ask their students to bring objects made up from different shapes to their classes; for example, a milk packet (rectangle) and Toblerone chocolate box (triangle). These objects brought from home are concrete manipulative materials that the children themselves are familiar with, and thus facilitate an introduction of shapes leading from concrete to abstract (recognizing shapes from the exercises in their textbooks and their workbooks).

Rationale and Purpose of the Study

Clements et al.'s (1999) study involved children in the United States. The purpose of this current study, in light of the results of the study by Clements et al., was to determine the criteria used by Primary One and Primary Two children in Singapore to distinguish members of a class of shapes; namely circles, squares, triangles and rectangles. It also aimed to find out whether or not there was evidence of a prerecognitive level before the van Hiele Level 0; the Visual Level. Finally, a further purpose of the study was to compare the results obtained between the six-year-old American children in Clements et al.'s study and Primary One Singaporean children.

Research questions

The following questions were thus formulated:

1. What criteria (visual or/and property) do Primary One and Primary Two pupils use to distinguish members of a class of shapes (namely, circles, squares, triangles and rectangles) from other shapes?
2. What criteria (visual or/and property) do Primary One and Primary Two pupils use to distinguish members of a class of shapes, namely circles and squares, from an embedded figure of overlapping geometric shapes?
3. Based on the above criteria, is there evidence that there is a prerecognitive level before the van Hiele Level 0 (Visualization)?
4. Is there a difference between the results of the Primary One Singaporean children and the six-year-old American children as studied in Clements, Swaminathan, Hannibal, and Sarama's (1999) research?

Significance of the study

To date research in geometry is still in the process of establishing "a consistent pattern of development on which to base instructional programs" (Hannibal, 1999). One difficulty appears to be that young children at different van Hiele levels of geometric thought interpret shapes in different ways. We do not have much data or information on young children's development in terms of the van Hiele levels of geometric thought. It is thus anticipated that the findings of this study will provide some insights into young Singaporean children's understanding with respect to basic shape recognition.

Research Design and Methodology

Subjects

A total of 40 students from a neighbourhood school participated in this study. There were 20 Primary One students, aged 6.5 to 7 years, and 20 Primary Two students, aged 7.5 to 8 years. These students were randomly chosen from two Primary One classes and two Primary Two classes. Thus, there was a mixture of high, middle and low ability group students in this group. Each of the four classes had a different mathematics teacher.

The Primary One students were studied just after their first semester examinations. The topic on geometry was introduced just before these examinations and hence when the students were tested, they had already completed the Primary One geometry unit. The Primary Two students were studied just before their second semester examinations, after they had completed the Primary Two geometry unit.

Interview

Data were collected through interviews in a one-on-one setting. The interviewer briefed the children on what they were expected to do before the shape-selection task and interview were conducted. Their responses in each interview were audio taped.

Students were asked to perform pencil-and paper five shape-selection tasks in which they were asked to "put a tick" on each of the shapes (circles, squares, triangles, rectangles and an embedded figure consisting of overlapping circles and squares respectively) on an A4 size page of geometric figures (see Appendix). The interviewer then asked the follow-up questions in order to determine the criteria the students used in making their selections. As with Clements et al.'s (1999) study, the focus of the interview was on the children's responses as they performed shape-selection tasks.

Instrumentation

Two sets of data were collected. The first set of data comprised students' scores for correctness. The second set of data consisted of the students' responses to the open-ended questions by the interviewer. The students' verbal responses were coded under two main criteria – visual and property (see Table 1). The codes were similar to those of Clements et al.'s (1999) study.

In cases of multiple verbal responses by the student, the dominant verbal response for each shape selection task was coded. Otherwise, the response was coded under "Multiple" response. Also, to be consistent with the van Hiele theory, a visual response followed by a property response was coded on the property level.

The verbal responses to each shape-selection task that were coded under the main criteria – visual - were further categorized into two groups, examples and non-examples of each class of shape. The same was done for the verbal responses to

Table 1
Coding of verbal responses of children

Category	Verbal Responses
Visual	Draws – Draws on paper or in the air, saying, "It looks like this."
	Looks like (shape) – Says "It looks [doesn't look] like a [shape name]."
	Another shape on page – Reference to another shape on same page, says "Same as this one.
	"Sort of like" – Says "Sort of like a [shape name]."
	Not the same as – Identifies another shape on page and declares, "This is not the same as that."
	Slanty/diagonal/bent – Visual references to lines that are not horizontal or vertical, says, "It's slanty."
	Pointy corners – Says "Pointy" or "It has corners."
	Looks like (object) – Says "Looks like a [object name]."
	Skinny/fat/long – Says "It's skinny/fat/long."
	Reference to size – Says, "It's big." Or "It's "small."
	Orientation – Gesturing along the diagonal of Shape 11 on the triangle task, saying "Its too bent down."
	Miscellaneous – Visual: Says, "It's a crazy one."
	Multiple – More than one visual response.
Property	Presence attribute – States and indicates by gesture presence or absence of specific attribute or property (e.g., Says "It's a rectangle because it doesn't have a point on the top.")
	Round/no sides – Round/curved/no straight lines/no corners (e.g., Says "It has a round top, so it's not a triangle.")
	Number of corners – Number of corners/angles.
	Number of sides – e.g., Says, "A triangle has three sides."
	Type of line – e.g., Says "It has two lines going up and down. " or "It (referring to base of triangle) is straight."
	Length of side – e.g., Says, "Two of the sides are the same length and the other two are the same length."
	Multiple – More than one property response.
IDK/NR	No response from child. Or says, "I don't know." Or "Just because"

each shape-selection task under the main criteria – property. In the case of examples, the children were asked to explain verbally why the shape was the required one. The same was done for the case of the non-examples.

One additional coding note: the Primary One mathematics syllabus states "To teach a square as a special case of a rectangle is premature". Thus students can treat squares and rectangles as different shapes. In this study, children who regarded squares as different from rectangles in the rectangle-selection task were considered as having chosen the shape correctly.

Also, for the triangle selection task, Shape 10 is made up of two triangles and Shape 12 comprises five triangles. Since it is possible that Shape 10 (for example) would not be chosen as it is not a triangle (but made up of two triangles), determination of whether both Shape 10 and Shape 11 are chosen correctly was determined by the reason given by the child.

Data analysis

To answer the first two research questions, the mean score and standard deviations were calculated for the five shape selection tasks. The percentage of all responses made by the children in the grade level (Primary One or Primary Two) in the given category was also calculated. To answer the third research question, in accordance with Clements et al.'s (1999) study, evidence to show that a prerecognitive level exists before Level 0 (Visualization) came from children's responses whether or not they could reliably distinguish the various shapes, namely circles, squares, triangles and rectangles from the non-examples of those classes of shapes.

To answer the fourth research question, a z-test was performed on the mean score of the Primary One Singaporean children and the six-year-old American children in Clements et al.'s (1999) study. The z-test was used instead of the t-test due to the large sample size; that is, the sample size was more than thirty (Ferguson & Takane, 1989, p. 171). With μ_1 and μ_2 the mean scores of the Primary One Singaporean children and the six-year-old American children, respectively, the null hypothesis, H_0, was $\mu_1 = \mu_2$ and the alternate hypothesis, H_1: $\mu_1 > \mu_2$. The alternate hypothesis H_1 is chosen due to the better TIMMS (Third International Mathematics and Science Study) geometry scores of the Singaporean children compared to their American counterparts. Using a one-tailed z-test at the 5% level, the null hypothesis H_0 was rejected if $z > 1.645$.

Results, Analysis and Discussion of the Data

Table 2 shows the number of children choosing a particular shape in the given category by each grade level (Primary One or Primary Two).

Table 2
Number of children choosing a particular shape in the given category by each grade level

Shape	Circles		Squares		Triangles		Rectangles		Embedded	
	P1	P2	P1	P2	P1	P2	P1	P2	P1	P2
1	0	0	19	18	19	14	3	2	1	0
2	20	20	1	1	1	2	0	0	17	17
3	20	20	5	1	6	2	16	13	3	0
4	0	0	20	20	15	5	0	0	19	18
5	20	19	12	7	8	2	5	8	20	18
6	0	0	1	1	18	14	17	17	3	1
7	20	20	0	0	6	2	0	0	3	0
8	1	0	20	20	18	20	6	10	18	11
9	20	20	19	17	9	6	20	20	10	8
10	1	0	19	20	20	13	16	14	1	8
11	2	0	11	6	12	6	0	0	1	8
12	19	20	0	0	18	15	20	20	1	8
13	20	20	11	6	11	6	1	2	1	8
14	20	19			6	3	12	9	5	1
15	20	20					1	0	3	3
16									6	0
17									8	2
18									19	20
19									1	0
20									1	0
21									3	4
22									12	19
23									20	16
24									20	20
25									0	1
26									20	19
27									17	17
28									18	19

The shaded portions sections reflect those shapes that are correct answers for the respective category. Notice, for example, in the embedded-figure test category, that almost all the Primary One children (18 of them) selected Shape 8 (square without horizontal base) which was a correct choice. However, only 11 Primary Two children chose this correct shape.

Research Question 1:
The following are the findings for the various shape-selection tasks.

Circle-selection task
The children generally had no difficulty in this task. Out of a possible score of 15, the mean scores for the Primary One children and Primary Two children were 14.75 and 14.95 respectively. The overall mean score was 14.85. The dominant response for examples of circles for both the Primary One and Primary Two children was "Round/no sides" (property response). Unlike Clements et al.'s (1999) study, Shape 11 (ellipse) was not the most distracting. There were more verbal responses for the non-examples of circles for both the Primary One and Primary Two children, likely as Clements et al.'s study suggested that "probably because it was easier for them to express differences from strong imaginistic prototype", that is, the visual prototype of a particular shape that was first introduced and taught to the children. Similar to Clements et al.'s study, holistic visual justification ("looks like circle") was most frequent for non-examples of circles.

Square-selection task
The children were quite able to discriminate a square from other shapes. Out of a possible score of 13, the mean scores for the Primary One children and Primary Two children were 11.25 and 10.7 respectively, with the overall mean score of 10.98. The dominant response for examples of squares for both the Primary One and Primary Two children was "number of sides" (property response). Twenty-five percent of the Primary One children identified the rhombus (Shape 3) as a square; in contrast to only 5% of the Primary Two children. Shapes 5, 11 and 13 are squares with no side horizontal. About slightly more than half of the Primary Two children compared to their Primary One counterpart's did not accept these shapes (5, 11 and 13) as squares. Both the Primary One and Primary Two Children gave more verbal responses for the non-examples of squares.

Triangle-selection task
Out of a possible score of 14, the mean scores for the Primary One children and Primary Two children were 10.2 and 11.55 respectively, with an overall mean score of 10.98. The dominant response for examples of triangles for both the Primary One and Primary Two children was "number of sides" (property response). More Primary One students correctly identified Shapes 1, 6, 7, 10, 11 and 12 as triangles; even though the mean score was lower than that for the Primary Two children.

In general, both the Primary One and Primary Two children who did not choose Shape 11 felt that it was too "long and skinny" to be a triangle. One reason for this observation could be that these children have a strong imaginistic prototype in mind and were thus unable to reliably distinguish triangles from the other non-examples.

Rectangle-selection task

Out of a possible score of 14, the mean scores for the Primary One children and Primary Two children were 11.1 and 11.4 respectively. The overall mean score is 11.25. The dominant response for examples of rectangles for both the Primary One and Primary Two children was that rectangles should be “long” (verbal response), thus a high percentage of children choosing Shapes 3, 6 and 10 as rectangles. Note that the children in Clements et al.’s (1999) study also tended to accept “long” parallelograms as rectangles. None of the Primary One and Primary Two children accepted Shapes 2 and 7 (squares) as rectangles. Again, the Primary One children gave more verbal responses for the non-examples of rectangles.

Research Question 2:

The following are the findings for the embedded-figure task:

Out of a possible score of 28, the mean scores for the Primary One children and Primary Two children were 20.65 and 22.6 respectively, with an overall mean score of 21.63. Again, the children gave more verbal responses for non-examples of circles and squares.

Embedded-figure: circles

For examples of circles, the dominant response for both the Primary One and Primary Two children was the property response “Round/no sides”. However, comparing the verbal responses given for this task to that for the circle-selection task, the percentage for the “Round” reason for examples of circles decreased. For non-examples of circles, also note the high percentage (65%) for the Primary One children saying “Looks like (circle)” as compared to the percentage (0%) for the same reason in the circle-selection task. Further, comparing to the circle-selection task, Shapes 6, 16 and 17 (ellipses) were distracting shapes in this embedded figure task. As mentioned earlier, ellipses were not distracting shapes in the circle-selection task.

Embedded-figure: squares

In Clements et al.’s (1999) study, 87% of the children chose Shape 22 (circle) and 35% of the children chose Shape 23 (square inside Shape 22). In this study, 78% of the children chose Shape 22 while 90% of the children chose Shape 23. It is interesting that only 80% of the Primary Two children chose Shape 23 as a square while 100% of the Primary One children chose Shape 23 as a square.

Research Question 3:
It is clear from the data that the children in the study were not able to reliably distinguish the examples of circles, squares, triangles and rectangles from their respective non-examples. For example, more children identified ellipses (Shapes 6, 14, 16 and 17) as circles in the embedded-figure task than in the circle-selection task. Clements et al.'s (1999) research also had this finding.

In general, children who scored full-marks for the circle-selection and square-selection tasks (5 Primary One children and 3 Primary Two children) did well for the embedded-figure task. The reader might want to take note that this was not the case for one particular Primary One child. That student scored the lowest score for this task from among the Primary Two students for the embedded-figure task even though he obtained full-marks for both the circle-selection and square-selection tasks.

Also, children who performed fairly well for the circle-selection and square-selection tasks did not necessarily do well in the embedded-figure task. A student scored the second lowest score for this task among the Primary One children even though he scored very well for the circle-selection and square-selection task. Similarly, there were children who performed well for the embedded-figure task but did not do so well in the square-selection task.

Both the Primary One and Primary Two children gave more verbal responses to non-examples than to examples. In addition, there was a discrepancy in reasons given for choice of shapes for the circle-selection, square-selection and embedded figure tasks. Therefore, as suggested by Clements et al. (1999), these children should be classified as prerecognitive. These children should not be classified as being in van Hiele Level 0 (Visualization) as they did not consistently classify the shapes based on the "looks like (shape)" criteria.

Thus there is evidence to support Clements et al.'s claim that a prerecognitive level exists before van Hiele Level 0 (Visualization).

Research Question 4:
Table 3 shows the comparison between the mean correctness scores and standard deviations between the 6-year-old American children in Clements et al.'s (1999) research and the Singapore Primary One children in my study.

It is clear from Table 3 that:

1. The six-year-old American children performed slightly better in the circle-selection and square-selection tasks than the Singaporean Primary One children.
2. Singaporean Primary One children scored better for the triangle-selection and rectangle-selection tasks.
3. Singaporean Primary One children performed slightly better than the American children for the embedded-figure task.

In answer to Research Question 4, z is approximately equal to 0.66 which is not greater than 1.645. Thus H_0 is not rejected and it is concluded that there was no significance difference between the American data and the Singaporean data.

Table 3
Mean scores of the six-year-old American children in Clements et al.'s (1999) research and the Singaporean Primary One children

Shapes	Possible Score	6-year-old American children	Singaporean Primary One
Circles	15	14.86 (0.4)	14.75 (0.7)
Squares	13	11.79 (1.7)	11.25 (1.4)
Triangles	14	8.48 (2.2)	10.2 (2.1)
Rectangles	15	8.79 (2.9)	11.1 (2.2)
Embedded	28	19.24 (2.4)	20.65 (3.5)

Further Discussion

In this study the children were not able reliably to identify shapes. They used different criterion (visual and property) for examples and non-examples of shapes. Why is such information important? Such "descriptions of children's early concepts of geometric shapes are important not only for theory but also for teacher education and for developers of constructivist-oriented curricula" (Clements et al., 1999). As teachers and educators, we should then ask ourselves why our children still have a discrepancy in criteria used to distinguish shapes despite the clearly specified objectives in the Singapore mathematics curricula. One reason possibly lies in our approach to introducing and teaching shapes.

Teachers typically use a visual-prototype approach. Many researchers and educators are concerned about the limited number of examples of geometric shapes used in schools. Clements et al. (1999) cautioned that such limited examples "impedes, and possibly undermines students' continued development of rich schemas for certain geometric shapes". Thus teachers should consider giving both examples and non-examples of geometrical shapes in their teaching.

Teachers need to listen to their students' understanding of shape as well. Sharp and Hoiberg (2001) felt that "from the moment each child enters a classroom, the teacher should begin listening for and recording clues to the child's van Hiele level". Teachers could then design their lesson to match their students' van Hiele level. In their attempt to raise their students' thinking to a higher van Hiele level, teachers themselves should also be encouraged to interview their own students. This enables teachers to have an even clearer view of their students' understanding of shapes, instead of relying on only written work and written tests (Sharp & Hoiberg).

Limitations and recommendations for further research

The results and findings are suggestive and should not be generalized to all the primary schools in Singapore. The results are also not conclusive due to the limited verbalizations of the young children involved in the study.

Future studies could examine Singaporean children's criteria used to recognize shapes using hands-on shapes. Children could be asked to do construction tasks like drawing shapes, instead of giving them shapes to mark out. A study could also be conducted to investigate the effects of orientation of shapes in young children's recognition skills. As well, another study could look into the effects of embeddedness of shapes on young children's shape selection criteria. There is evidence from my study that a prerecognitive level exists before Level 0 (Visualization). Thus how do we get children to move from this prerecognitive level to Level 0? Do the Phases of Learning apply to the prerecognitive level as well?

To echo Clements et al.'s (1999) concern, "research is needed to identify the specific, original intuitions and ideas that young children develop about figures". Indeed, there is still a need to have more research done with Singapore children's geometrical ideas and concepts.

Concluding Statement

With the above discussions in mind and the knowledge of how our young children recognize shapes, it is necessary for insightful educators, teachers and designers of curriculum to aim towards "a vision of high-quality early childhood education" (Clements, Sarama, & DiBiase, 2002, p.513). Only then can we not only provide our children with the best teaching but also the richest classroom mathematical experiences.

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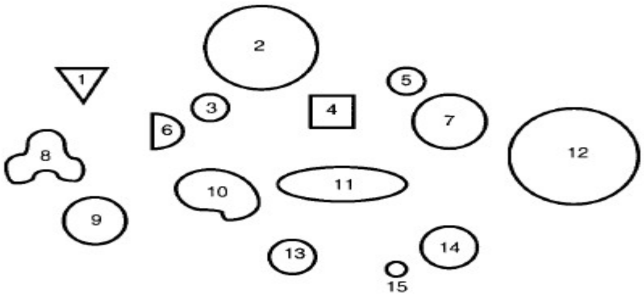
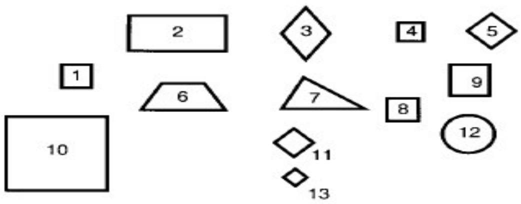
Appendix

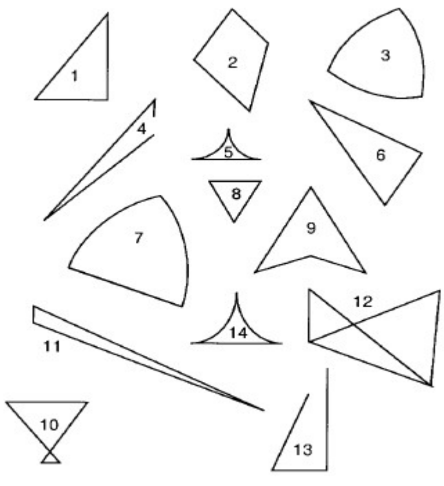
1. Student marks circles (Razel & Eylon, 1991).



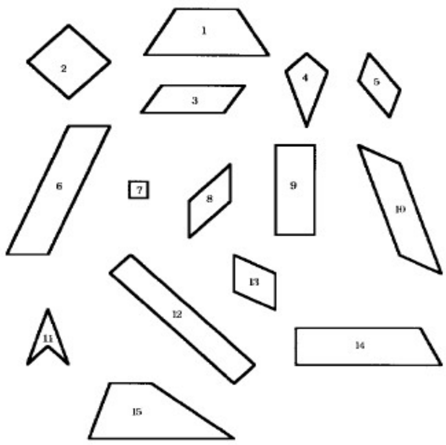
2. Student marks squares (Razel & Eylon, 1991).



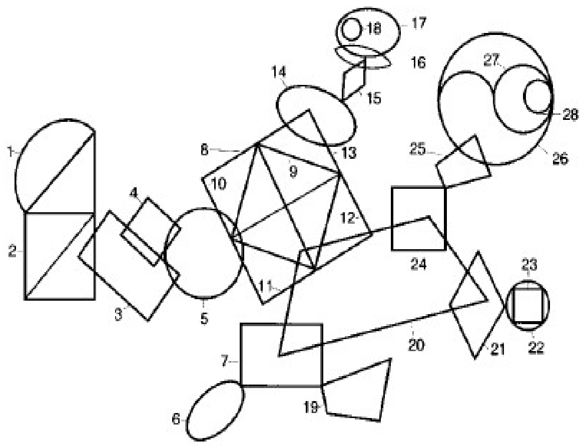
3. Student marks triangles (Burger & Shaughnessy, 1986; Clements & Battista, 1992a).



4. Student marks rectangles (Burger & Shaughnessy, 1986; Clements & Battista, 1992a).



5. Student marks first the circles, then the squares, with different colors (Razel & Eylon, 1991).



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